Mathematics as story

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We contend that mathematics is an aesthetic and a storied experience. In this paper we explore the interplay between what is an 'aesthetic mathematics experience' and a 'good mathematics story'. We do this in the context of a mathematical applet, namely the *Colour Calculator* (Sinclair 2001), and in the context of our research into the mathematical and pedagogical thinking of elementary mathematics teachers. In the end, we propose that it is perhaps redundant to use both 'aesthetic' and 'story' to describe the mathematics experience, as stories are naturally aesthetic in nature. First, however, we discuss what we mean by aesthetic and by story.

The aesthetic

The aesthetic is a historically and biologically verifiable human predisposition (Dissanakye 1992; Boyd 2001), a means by which humans make sense of the world (Egan 1997; Lakoff and Nunez, 2000) and an element of pedagogy and constructed environment (Eisner 1985, 1988; Greene 1995). Within this vision of the aesthetic, people are predisposed to their senses of rhythm and fit, balance, motion and symmetry. Dissanakye (1992) talks about an "aesthetic sensibility" that "acts as one of our primary meaning-making capacities in all domains" (25). Greene (1995) has characterized the aesthetic as an alertness, a "wide-awakeness". Greene says educators can learn from artists, whose way of comprehending experience is through perception, imagination, feeling and form. Aesthetic engagement has to do with being open to one's own sense of curiosity and intuition.

Labelle (2000), professor of mathematics at the Université du Québec à Montréal, says "I like mathematics because it is beautiful, full of surprises, and gives me complete freedom of thought." Feelings of surprise and beauty are emotional. Doing mathematics is emotional. Students also express these attributes. "Math is just another way of both creating meaning and describing it. It's lovely. I'm lousy at it, but I love feeling my brain tumble over as it understands something for the first time." The feeling of connectedness to stimuli that math provides this professor and student should not be mistaken for sentiment or mere personal expression. Lakoff and Johnson (1999, 176) state that emotion is "inextricably linked to perception and cognition" and "is better understood as the tension or excitement level produced by the interaction of brain processes of perception, expectation, memory and so forth".

Mathematics is an aesthetic engagement when we give our attention to doing mathematics. Attention is aesthetic in nature (Davey 1999, 25). Whenever we bring consciousness to bear on a topic, either individually or communally, we engage it in emotional and imaginative ways. We use attention to learn and extend ourselves, to incorporate a new thing, whether that is a new way of solving an old problem or finding news ways to express an idea (Gadanidis, Hoogland & Hill 2002a, 2002b). We extend our understanding, we become more complex, and this feels good. In Dissanayake's (1992) words, "as *Homo aestheticus*, we really require beauty and meaning." Attention is our way of gaining beauty and meaning from experience.

Story

Human cognition is story based. We think in terms of stories, we understand the world in terms of stories that we have already understood, we learn by living and accommodating new stories and we define ourselves through the stories we tell ourselves (Schank 1990; Bruner 1990, 1996). We "live most of our lives in a world constructed according to the rules and devices of narrative" (Bruner 1996, 149). We are actors in real-life stories where "we walk on stage into a play whose enactment is already in progress – a play whose somewhat open plot determines what parts we play and toward what denouements we may be heading" (Bruner 1990, 34). Our lives make sense when shaped into narrative form (MacIntyre 1984, 39).

Story is a human symbol system used to comprehend events and entertain questions, and represent those events and questions in multi-modal ways. Multi-modal refers to story's ability to hold the complexity of experience in a single work, and to present sensory, emotional and conceptual information simultaneously through image and metaphor. By embodying the complexity that we seek to experience, good stories offer the pleasure of new insight, of more complex ways of understanding our world. Story naturally incorporates the aesthetic. McKee (1997, 237), writing about what constitutes a good story, states: "Insight is the audience's reward for paying attention, and a beautifully designed story delivers this pleasure scene after scene after scene." Bruner (1964, 18) states that an act that produces insight is "the hallmark of a creative enterprise." A story also has its own internal system of validation in that it is not a story unless it coheres, and complies with its own internal logic. In this way story forces people to concern themselves with the logic of their stories, with making sense of their experience. Thus story provides its own justification, or *proves* itself. This cohesion of ideas and events creates meaning and pleasure.

Dewey (1938) said that although “all genuine education comes about through experience” (25), not all experiences are educative. He also said that educational experiences are those “that live fruitfully and creatively in subsequent experiences” (28). We conceptualize an educative mathematics experience as a good mathematics story. Schank (1990) suggests that not all experiences make for a good story. Good stories are often about things that are not routine-like and involve some element of surprise and they are of personal interest and significance to the person hearing or living the story. Good stories trigger pleasurable reactions as they offer us opportunities “to use our minds in fresh, experimental ways, to flex our emotions, to enjoy, to learn, to add depth to our days” (McKee 1997, 5). Setting, rising action, with some sort of ‘conflict’, and the resolution of conflict are integral components of good stories. Good stories represented as graphs would have peaks and valleys.

In contrast, the mathematics story that is experienced in school is often flat-lined. It does not have the peaks and valleys of a good story, it does not engage student’s mathematical attention, and it offers minimal opportunities for experiencing mathematical insight. The focus is often on mastering mathematical procedures and getting correct answers rather than investigating mathematical relationships. The traditional mathematics classroom story is typically not a good story to tell, it is not an interesting story to live through. It is not a story that lives fruitfully in future stories.

We are not suggesting that when a teacher facilitates a good mathematics story all students experience it as such. Likewise, students can and do have aesthetic mathematics experiences in poor mathematics stories. Stories are not top-down experiences – they are interactive experiences. Unlike typical actors on a stage, who follow a set script, students in the classroom are actors that can and do write their own scripts. However, the scripts they can write and act out in the classroom are greatly affected by the mathematics story facilitated by the teacher. So a good mathematics story is good in that it increases the potential for students to engage aesthetically with mathematics. Mathematics as story is about potential roles rather than prescribed roles for students.

The aesthetic within mathematics story

Story is an ideal embodiment of the aesthetic. Story acts as a stage, a setting or a context, which allows the aesthetic free reign. We conceptualize aesthetic mathematical engagement as an integral feature of the good mathematics story. The key components of this conceptualization include the following: (1) we naturally seek the pleasure of coming to understand, of becoming more complex by gaining insight from our experience, (2) we gain insight through attention, and attention is aesthetic in nature, (3) aesthetic mathematical engagement occurs in good mathematics stories, stories which draw our attention and offer us the pleasure of mathematical insight, and (4) good mathematics stories are regenerative, as they live fruitfully in future mathematics stories.

In the following section, we apply this conceptualization of mathematics as story in an analysis of the *Colour Calculator* applet (see Figures 1 and 2). The *Colour Calculator* has been researched in terms of the relevance of the aesthetic and gives us a context for comparing and contrasting our own view of the aesthetic in mathematics. The *Colour Calculator* also leads us into a discussion of the extent to which it offers the potential for a good mathematics story.

The *Colour Calculator*

Although it seems to us obvious that the aesthetic is relevant in mathematics education, the aesthetic also seems to be elusive when attempting to purposefully incorporate it in mathematical experience. Since the aesthetic is typically associated with the arts, at first it may seem that we should search for arts-mathematics links. Sinclair (2001, 25) cautions that traditional uses of artistic connections with mathematics, such as “modular arithmetic in music or regular polygons in eurythmy”, may actually “undermine the aesthetic” in mathematics. “A pernicious consequence of appealing to students’ love of something else (whether in the arts, sports, food, or money) in the hopes of increasing their interest levels in mathematics is that it endorses the belief that mathematics itself is an aesthetically sterile domain.” Sinclair suggests that we need to “reverse the direction of the aesthetic flow, so that it originates in the mathematics” (25). Sinclair defines aesthetically rich mathematical experiences as experiences that enable us to “wonder, to notice, to imagine alternatives, to appreciate contingencies and to experience pleasure and pride” (26).

The *Colour Calculator* is an attempt to create an aesthetically rich mathematics environment for learning about fractions and decimals. It is a web-based calculator that displays 100 decimal digits and also displays colour representations of the decimal digits of results in table form. For example, when calculating 1/13, the *Colour Calculator* will display a result similar to the one shown in Figure 1 (the difference being that the *Colour Calculator* shows all 100 decimal digits). The user has the option of changing the width of the table, as shown in Figure 2.

What is it about the *Colour Calculator* that, compared to a regular calculator, enhances aesthetic engagement? When showing the result to 1/13, a regular calculator may display 0.0769230. By displaying 100 decimal digits the *Colour Calculator* makes it possible to notice that the sequence of decimal digits 076923 repeats. The *Colour Calculator* also displays the 100 digits as colours (each of the 10 digits 0-9 is represented by a different colour) in a rectangular table (where the user has control of the width of the table). The goal is to transform the “often abhorred” (28) topic of fractions and decimals into a sense making of mathematical ideas that uses some of students’ “aesthetic sensitivities such as symmetry, repetition, rhythm and pattern” (28). There is an implication that the incorporation of colour to represent or replace numbers is primary in the aesthetic qualities of the *Colour Calculator.* This is suggested by the name given to the calculator and by contrasts made between "manipulating fractions and decimals as colour patterns” and manipulating them “as 'ugly numbers'" (27).

We suggest that the primary aesthetic feature of the *Colour Calculator* is not the use of colour but rather the table organization of the 100 decimal digits. The use of colour, while it helps make existing patterns more visible and more pleasing to the eye, does not help students see new patterns. It is the table format that is the source of new pattern generation. For instance, suppose we removed colour from the table representation, as shown in Figure 3. Although the patterns in the table are less pronounced without colour, they are still visible – and sensed in the same way that one’s eye is drawn to a vertical, diagonal or other visible character repetition pattern in a page of text. We would also argue that there is pleasure in seeing a pattern in the table of numbers that is not immediately apparent (as it would be with colour added) – in that the challenge of seeing the pattern makes the seeing of the pattern more rewarding. In addition, the replacement of digits with colours reduces the pattern options that students may attend to, especially ones that deal with operations with digits in the table.

We also wonder about the effect of replacing the digits in the table with colour on students' perception of the source of the aesthetic in mathematics, of the pleasure of sensing patterns. Certainly the source of the colour pattern in the *Colour Calculator* is mathematical in the sense that it is a representation of a numeric pattern. However, the replacement of numbers with colour may give students an incorrect message about the source of the aesthetic – that it is colour that makes numbers pleasing and not the sensing and exploration of pattern in the numbers themselves. We suggest that the *Colour Calculator* may be improved in this respect by offering colour as a choice rather than as a default and by using colour to shade the background of digits rather than replace them, as is shown in Figure 4. As students use the Show Colour feature to activate the background colours of say the digits 0 and 6 (as shown in Figure 4), the background colour of these digits is filled in in the table. Notice that the digits remain visible so that students may explore patterns that may involve numeric operations. The incorporation of colour as an option has the advantage of increasing both student control and the possible patterns that may be explored. It also incorporates colour without reducing the emphasis on numbers. A further feature may be the option of a gray scale to represent the digits 0-9, with the darkness of the shading increasing with the digits' magnitude. Colour is a substitutive representation and hides the relative size of numbers, whereas shading is an additive representation and maintains a sense of the relative size of numbers (Norman 1993, 69).

The *Colour Calculator* (Figure 1) and the revised *Colour Calculator* (Figure 5) improve the typical school experience with fractions and decimals in that they provide the context for engaging students "in a whole, unified and intrinsically satisfying experience" (Sinclair 2001, 31). This stands in contrast to typical teaching and learning about decimals and fractions where the starting point is rules, definitions and teacher demonstration of procedures rather than student exploration and sense making. It is interesting that in both cases, the traditional approach and the *Colour Calculator approach*, students may be exposed to the same concepts and ideas, such as the same patterns in repeating decimals, yet the aesthetic potential differs significantly. We suggest the difference is that the *Colour Calculator* increases the potential for a better mathematics story, by creating a patterning environment that draws student attention and facilitates the experience of mathematical surprise and discovery, of mathematical insight. A traditional classroom approach relies on shortcuts to 'mathematics story' development, being more like “the pamphlet version of discovery with numbered steps” (Gillard 1996, xi) where in the rush to conclusions and rules, we miss the pleasure of the process, of the journey.

The *Colour Calculator* is a good mathematics story. It offers a level of complexity to the study of fractions and their decimal forms that draws our mathematical attention, and allows us opportunities to notice, to imagine, to explore, and to gain mathematical insight. In the next section we discuss aesthetic mathematical engagement and the good mathematics story in the context of teachers’ personal views and experiences of mathematics.

Teachers and mathematics story

In our research of critical mathematics teacher education experiences (Gadanidis & Hoogland 2002; Gadanidis, Hoogland & Hill 2002a, 2002b) – experiences that help teachers see mathematics and mathematics teaching in new light– we have encountered the aesthetic in four different places: (1) In teachers’ views of mathematics illustrated through their stories of mathematical experiences; (2) in teachers’ engagement with mathematics in teacher education settings; (3) in moments of insight associated with doing mathematics; and (4) in pre-service teachers’ reactions to mathematicians’ expressions of affection towards mathematics. Our research has been primarily focused on the experiences and thinking of teachers taking fully online mathematics teacher education courses.

Teachers’ views of mathematics

Teachers were asked to share their views of mathematics and to explain some of the reasons or sources for their views. We found it interesting that teachers responded by telling us stories – rather than make a list of characteristics. Most teachers in the online course expressed a negative reaction to mathematics, as manifested in statements like “I grew up with very negative feelings towards math.” In addition, some teachers expressed a passive attitude towards the subject. There wasn’t a feeling of dislike or fear but neither was there a feeling of excitement or enthusiasm. As one teacher commented, “I did not hate math in school but I didn’t love it either. I went through the motions.” It was interesting to observe that although negative and passive stories about mathematics were primarily school based, positive stories of mathematics were often family based. As one teacher related, “I, like many of you, had many problem-solving car trips. I still get excited when I see a license plate that I can make ten with (using any means).” This is not to say that all home experiences of math are positive and school experiences are negative, rather, that in our study mathematical activity in the home was remembered fondly and its pleasurable affects continued to be felt in adult life. Although negative and passive stories focused on learning procedures and getting answers (or the difficulty in getting correct answers), positive stories focused on problem-solving processes. For example, note the aesthetic qualities of the earlier open-ended mathematical problem of "making ten" using "any means" which contrasts sharply with the more traditional approach that insists that one finds *the answer,* to, say, 5 + 5. The person "making ten" has the opportunity to use her imagination and to find personal, creative ways of looking at mathematically combined digits on license plates.

Teachers' mathematical engagement

In addition to relating stories of past mathematical experiences, teachers in the online course experienced stories of mathematics that were different from typical school mathematics. Interestingly, in contrast to the *Colour Calculator* which sought to replace numbers with colour, the first mathematics activity focused solely on operations with numbers. Teachers were asked to mentally solve 16 x 24 or 156 + 78 + 9, to record the mental processes they used, and to post them online along with a comparison with the standard paper and pencil methods for solving these problems, and the possible implications for teaching and learning mathematics. The reason for focusing on number operations is that they have been traditionally at the centre of elementary mathematics education. Although current curriculum documents are expanding the scope of the mathematics curriculum, numeracy remains one of the 'basics' of elementary mathematics that teachers, and for that matter parents and the public in general, emphasize above all other areas of elementary school mathematics. In this first mathematics experience the goal was to involve teachers in doing and thinking about typical mathematics problems in new ways – especially mathematics problems that are usually associated with negative aesthetic experiences, like number operations. Although mathematics activities with richer contexts were used in later parts of the course, the intent was to give teachers an early experience where the context would not be a distraction. For example, had the course started with a mathematics activity that used the *Colour Calculator,* teachers may have associated any positive aesthetic reactions with either the use of colour or the use of the *Colour Calculator* itself, rather than with their exploration of number-based patterns.

We suggest that the experience of mentally solving 16 x 24 or 156 + 78 + 9 offers aesthetic mathematical engagement in that the mental processes involved do not demand rule-based procedures. How people solve 16 x 24 depends greatly on how they personally interpret the problem. For example, some people may multiply 16 and 25 and then subtract the extra 16. Others may deconstruct the problem as 10 x 24 + 6 x 24. Many other solutions processes are possible – even ones that use algebraic structures like (20 - 4)(20 + 4). Given such problems, people are eager to share their solutions, they express interest and sometimes surprise in the solutions of others, and are motivated to try to come up with different solution processes. Open-ended inquiry, interest, surprise and motivation are characteristics of an aesthetic approach. Teachers need such personal experiences with mathematics, where they give their attention to doing mathematics and gain mathematical insight, if they are to create similar stories in their classrooms.

Teachers' mathematical and pedagogical insights

Teachers noticed that their mental solution processes for 16 x 24 and 156 + 78 + 9 differed from the standard algorithms for addition and multiplication, and that several different solution processes emerged: "When I added 156 + 78 + 9 I started with 156 + 9 = 165. Then I added 165 + 80 = 245 - 2 = 243. This is different than when I did it with paper and pencil." And many teachers commented that typically their mental addition methods operated on digits in a reverse sequence compared to the paper and pencil method: "I solved my problem by starting with the bigger numbers first (left to right, not right to left!)". For most teachers, the experience of solving addition and multiplication problems mentally provided new insights into mathematics. They were at once surprised and pleased to see that mathematics can be done in flexible and personal ways, that these ways are easy to understand, and that they are interesting to share and compare and to consider alternatives. It may seem unlikely that teachers who have teaching experience, and in some cases lengthy experience, with addition and subtraction would be surprised by their mental mathematical processes. Yet this phenomenon has been observed when similar activities have been provided by one of the authors in numerous professional development setting for teachers and mathematics sessions for parents. There is an electric atmosphere in the room when teachers and parents share personal solution methods and a sense of epiphany about what mathematics is or what it could be.

As is the case with the *Colour Calculator*, the experiences related to mentally solving 16 x 24 or 156 + 78 + 9 are good mathematics stories to live through. From a teaching point of view, they provide teachers with first hand experience of mathematics that is not rule-based and also not answer-based. Although teachers do experience pleasure in getting correct answers using their personal mental methods, the greatest pleasure is experienced in the sharing and comparing of methods and in realizing that mathematics answers can be derived using a variety of personal, informal and flexible solution processes. Mathematics becomes more of a set of interesting and connected personal stories to be shared and discussed, rather than a set of procedures to be memorized and applied. And the focus shifts from memorizing to understanding. "To me, the implications are that doing arithmetic mentally requires real understanding. The traditional way (on paper, doing the "ones" first) is more of a procedure to be memorized that requires little understanding". Without such personal experiences with mathematics, it is difficult for teachers to imagine what mathematics reform documents mean when they refer to investigation or communication. In fact, such documents may be imagined as inkblots where teachers see only what they are able to see – and it's difficult for a teacher to see what investigation and communication may mean in a mathematics classroom when they have not experienced it themselves.

> I feel like [this experience] has cleaned my spectacles […] seeing how different people calculate […] Hmph! It is not until I do something myself, do I more fully understand the language and what the curriculum is really driving at. Thus students need to explore different ways of doing calculations, talking about it, communicating their ideas in a variety of ways.

Perhaps more importantly, teachers may be more likely to move towards a mathematics teaching approach that involves investigation and communication because they have experienced pleasure with it.

It is not uncommon for elementary teachers to have negative aesthetic associations with mathematics. Many openly and sometimes proudly admit that they do not like mathematics or that they do not feel confident mathematically. Changing teachers' perceptions of mathematics is an important first step towards improving classroom practice. McGowen & Davis (2001a) suggest that we need an "antidote" to teachers' conceptions of mathematics as learning procedures and getting right answers. Findings show that such conceptions are consistently associated with observed practice of teachers (McGowen & Davis 2001a, 2001b; Stipek et al

2001) and that teachers who hold such conceptions of mathematics have lower teacher self-confidence and enjoy mathematics less than teachers who hold inquiry-oriented conceptions (Stipek et al 2001). We suggest part of the ‘antidote’ may be teachers experiencing and internalizing new mathematics stories – new learning and teaching roles – where their attention on mathematical ideas leads to the pleasure of mathematical insight.

Teachers’ reactions to mathematicians’ views

In one of our research projects we are looking at pre-service elementary teachers’ reactions to interviews with mathematicians where mathematicians express affection towards mathematics. Short excerpts from three interviews (Labelle 2000; Sinclair 2000; Yeats 1999) read and discussed by the pre-service teachers are shown in Figure 6. It is interesting that most of the sixty pre-service teachers in the study – most of which entered the faculty of education experience with very negative attitudes towards mathematics – expressed positive aesthetic reactions to the mathematicians’ views of mathematics and they shared personal examples of mathematics experiences that they found aesthetically pleasing. Listed below are excerpts from two pre-service teachers’ reactions to the mathematician’s views. Similar reactions were expressed by many of the other pre-service teachers in the study.

> My initial response to the question regarding the beauty of mathematics was one of disbelief! I honestly never considered such an adjective as applicable to the subject of math. Yet with a little introspection I can remember …

> After reading the interviews with the mathematicians, particularly Nathalie Sinclair's, I felt a creeping desire to tackle math again. The language that she used to describe her love of math was inspirational, as well as, poetic, appealing to my senses, and the possibility of actually loving math again. ... Maybe it is possible, after all, to enjoy math again. We'll see …

We believe that teachers naturally want to improve the mathematics stories they possess, live and help create in their classrooms. They are aesthetically drawn to a good story and they naturally want to find ways of incorporating it in their lives by retelling it and reliving it and by improving the mathematics story they have internalized – the story they tell to themselves.

Mathematics as story

Thinking of mathematics as story provides a number of interesting dimensions to mathematics. First, as we have argued earlier, mathematics as story naturally incorporates the aesthetic. This binds the aesthetic to considerations of what constitutes a good mathematics story. Second, story adds a directional aspect to mathematics experience. Stories we experience live in our future experiences and help direct them. Good mathematics stories that we have internalized are useful in drawing our attention to considerations of what type of mathematics story lives fruitfully in future mathematics experiences. Third, mathematics as story can better capture the complexity of experience than say a list of characteristics. Fourth, viewing mathematics as story can help communicate mathematics reform directions. Teachers naturally think in terms of stories, especially in the case of elementary teachers who are likely to have a strong language arts background. Lastly, we also believe that teachers naturally have a sense for judging the quality of a story, and this can be a guide for planning mathematics activity that is a good story for their students to experience.

Mathematics as story raises a number of interesting further research questions. First, how exactly is the aesthetic bound within mathematics story? This paper provides a starting point for considering this question. Second, are some story genres more appropriate in describing the mathematics experience? For example, is mathematics as story best viewed as a romantic story? Our research in mathematics teacher education suggests that this may be a good fit, as our analysis has uncovered the idea of romantic story with its ideas of overcoming difficulty and finding happiness (or wholeness) at the end of the day (Gadanidis, Hoogland & Hill 2002a). Stories of how teachers come to love math, or hate it, and in some cases, how an old passion is sparked and reignited may be useful research topics and may offer professional development direction. Third, how useful is the concept of story for teachers in terms of their ability to judge and improve the quality of the mathematics experience they are providing for students? For example, how useful would it be to ask teachers to tell stories of teaching and learning in their classrooms and then analyze them in terms of what makes a good story? Fourth, would story be a better tool for communicating mathematics education reform? Currently, reform documents and teaching resources rely heavily on lists and categories of the characteristics of mathematics experience, which are not good communication tools. Would incorporating story formats in curriculum documents or teaching resources help teachers better understand the mathematics reform direction? Lastly, what is the interplay between various stories of mathematics – the stories we have internalized through past experiences, new stories we live through, stories others tell, and stories we tell ourselves?

Our research supports the view that the aesthetic is relevant in mathematics education, as in our studies the aesthetic appears to be an integral part of teachers' views of mathematics, of moments of epiphany and in reactions to rich mathematics experiences and to mathematicians' expressions of affection towards mathematics. We posit that mathematics as story may provide a construct that naturally incorporates the aesthetic while at the same time captures the context and complexity of mathematics experience. Story is a natural form of thinking and communication and provides a meaningful way of talking and thinking about teaching and learning mathematics.

References

Boyd, B. (2001). The Origin of Stories: Horton Hears a Who. *Philosophy and Literature, 25*(2), 197-214.

Bruner, J.S. (1964). *On Knowing. Essays from the Left Hand*. Cambridge, MA: Harvard University Press.

Bruner, J.S. (1990). *Acts of Meaning.* Cambridge, MA: Harvard University Press.

Bruner, J.S. (1996). *The Culture of Education*. Cambridge, MA: Harvard University Press.

Dewey, J. (1938). *Experience and education*. New York, NY: Collier Books.

Dissanakye, E. (1992). *Homo aestheticus*. New York, NY: Free Press.

Egan. K. (1997). *The educated mind: How cognitive tools shape our understanding.* Chicago, Illinois: The University of Chicago Press.

Eisner, E. (1985). Aesthetic modes of knowing. In Eisner, E. (ed.) *Learning and teaching the ways of knowing, (Eighty-fourth Yearbook of the National Society for the Study of Education)*. Chicago. IL: University of Chicago Press, 23-36.

Gadanidis, G. & Hoogland, C. (2002). Mathematics teacher education online. *Proceedings of the World Conference on Educational Multimedia, Hypermedia and Telecommunications*, Denver, Colorado.

Gadanidis, G., Hoogland, C., & Hill, B. (2002a). Mathematical romance: Elementary teachers' aesthetic online experiences. *The 26th Conference of the International Group for the Psychology of Mathematics Education*, University of East Anglia.

Gadanidis, G., Hoogland, C., & Hill, B. (2002b). Critical experiences for elementary mathematics teachers. Proceedings of the XXIV Annual Meeting, North American Chapter of International Group *for the Psychology of Mathematics Education* (in press). Athens, Georgia.

Gillard, M. (1996). *Story teller story teacher – Discovering the power of story telling for teaching and living*. York, Maine: Stenhouse Publishers.

Greene, M. (1995). *Releasing the imagination: Essays on education, the arts and social change*. San Francisco, CA: Jossey-Bass.

Kamii, C., B.A. Lewis, & S.J. Livingston (1993). Primary arithmetic: Children inventing their own procedures. *Teaching Children Mathematics, (1)*9, 77-99.

Labelle, G. (2000). Interview with Gilberte Labelle. *MathMania, 5*(4), 10-11.

Lakoff, G and Nunez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books.

McKee, R. (1997). *Story – Substance, structure, style, and the principles of screenwriting.* N.Y.: Harper-Collins/Reagan Books.

McGowen, M.A. and Davis, G.E. (2001a). What mathematics knowledge do pre-service elementary teachers value and remember? In R. Speiser, C.A. Maher, & C.N. Walter (eds.) *Proceedings of the XXIII Annual Meeting, North American Chapter of International Group for the Psychology of Mathematics Education* (pp. 875-884). Snowbird, Utah.

McGowen, M.A. and Davis, G.E. (2001b). Changing pre-service elementary teaches' attitudes to algebra. In H. Chick, K, Stacey, J. Vincent, & J. Vincent (eds.) *Proceedings of the 12th ICMI Study Conference: The future of the teaching and learning of algebra* (pp. 438-445). Melbourne, Australia: University of Melbourne.

McIntyre (1984). *After virtue: A study in moral theory.* Notre Dame, Indiana: University of Notre Dame Press.

Norman, D.A. (1993). *Things that make us smart: Defending human attributes in the age of the machine.* NY: Addison-Wesley Publishing Company.

Schank, R. (1990). *Tell me a story – A new look at real and artificial memory.* N.Y.: MacMillan Publishing Company.

Sinclair, N. (2000). The joy of mathematics. *MathMania, 5*(3), 4.

Sinclair, N. (2001). The aesthetic is relevant. *For the Learning of Mathematics*, *21*(1). 25-32.

Stipek, J.S. et al (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213-226.

Yeats, K.A. (1999). Interview with a mathematician. *MathMania, 4*(1), 6.

Figures 1-5

Equation: 1/13
Results: .076923076923076923076923076923076923076923076923076923076923 …

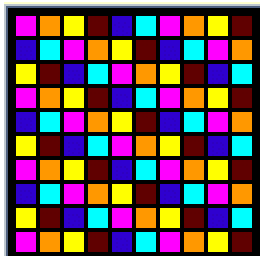

Figure 1. 1/13 as displayed on the Colour Calculator (table width set to 10)

Equation: 1/13
Results: .076923076923076923076923076923076923076923076923076923076923 …

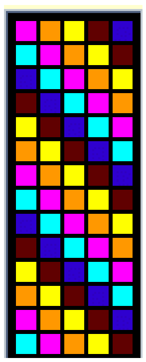

Figure 2. 1/13 as displayed on the Colour Calculator (table width set to 5)

Equation: 1/13
Results: .076923076923076923076923076923076923076923076923076923076923 …

0	7	6	9	2	3	0	7	6	9
2	3	0	7	6	9	2	3	0	7
6	9	2	3	0	7	6	9	2	3
0	7	6	9	2	3	0	7	6	9
2	3	0	7	6	9	2	3	0	7
6	9	2	3	0	7	6	9	2	3
0	7	6	9	2	3	0	7	6	9
2	3	0	7	6	9	2	3	0	7
6	9	2	3	0	7	6	9	2	3
0	7	6	9	2	3	0	7	6	9

Figure 3. 1/13 displayed in table form without colour

Equation: 1/13
Results: .076923076923076923076923076923076923076923076923076923076923 …

0	7	6	9	2	3	0	7	6	9
2	3	0	7	6	9	2	3	0	7
6	9	2	3	0	7	6	9	2	3
0	7	6	9	2	3	0	7	6	9
2	3	0	7	6	9	2	3	0	7
6	9	2	3	0	7	6	9	2	3
0	7	6	9	2	3	0	7	6	9
2	3	0	7	6	9	2	3	0	7
6	9	2	3	0	7	6	9	2	3
0	7	6	9	2	3	0	7	6	9

Show Colour
0
1
2
3
4
5
6
7
8
9

Figure 4. Redesigned Colour Calculator allowing user choice of colour

"I like mathematics because it is beautiful, full of surprises, and gives me complete freedom of thought."

Gilbert Labelle, professor of mathematics at the Université du Québec à Montréal

"I like math because it's beautiful, and because working with it is fun. I considered going into music composition, and I really feel the aesthetics of the two subjects are very closely related. The search for elegance."

Karen Amanda Yeats, mathematics student at the University of Waterloo

"Often mathematics is beautiful when it lets you see something in a new way or remind you of something else that you already know and enjoy. … When we experience things fitting together, they often look beautiful to us, and they often bring us a sense of pleasure."

Nathalie Sinclair, Queen's University

Figure 5. Excerpts from interviews with mathematicians.